

# Neutrino Oscillations With Two Sterile Neutrinos

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June 9, 2016

PACS Indices:11.30.Er,14.60.Lm,13.15.+g

Keywords: sterile neutrinos, neutrino oscillations, U-matrix

## Abstract

This work estimates the probability of  $\mu$  to  $e$  neutrino oscillation with two sterile neutrinos using a 5x5 U-matrix, an extension of the previous estimate with one sterile neutrino and a 4x4 U-matrix. The sterile neutrino-active neutrino mass differences and the mixing angles of the two sterile neutrinos with the three active neutrinos are taken from recent publications, and the oscillation probability for one sterile neutrino is compared to the previous estimate.

## 1 Introduction

Reviews of experimental data on neutrino oscillations[1, 2, 3] find that there probably are two sterile neutrinos. Refs. [1, 2] by Kopp *et. al.* discuss a variety of experiments on neutrino oscillations, with appearance and disappearance, while the present work treats  $\nu_\mu$  to  $\nu_e$  appearance. A recent analysis of neutrino oscillation experiments with one and two sterile neutrinos[4] estimate the sterile neutrino masses and mixing angles used in the present work.

In the present work we use a U-matrix approach, introduced for active neutrinos with a 3x3 U-matrix[5], and extended to a 4x4 U-matrix with one sterile neutrino in a recent study of  $\mathcal{P}(\nu_\mu \rightarrow \nu_e)$ , the transition probability for a muon neutrino to oscillate to an electron neutrino[7, 8]. We introduce a 5x5 U-matrix for three active and two sterile neutrinos, using the parameters found in Refs.[1, 2] and Ref.[4]

## 2 5x5 U-Matrix

Active neutrinos with flavors  $\nu_e, \nu_\mu, \nu_\tau$  and two sterile neutrinos,  $\nu_{s1}, \nu_{s2}$  are related to neutrinos with definite mass by

$$\nu_f = U \nu_m, \quad (1)$$

where  $U$  is a 5x5 matrix and  $\nu_f, \nu_m$  are 5x1 column vectors. We use the notation  $s_{ij}, c_{ij} = \sin\theta_{ij}, \cos\theta_{ij}$ , with  $\theta_{12}, \theta_{23}, \theta_{13}$  the mixing angles for active neutrinos; and  $s_\alpha = \sin(\alpha), c_\alpha = \cos(\alpha), s_\beta = \sin(\beta), c_\beta = \cos(\beta)$ , where  $\alpha = \theta_{i4}, \beta = \theta_{i5}$  are sterile-active neutrino mixing angles, with  $i=1,2,3$ , and  $\delta_{CP}=0$ .

$$U = O^{23} O^{13} O^{12} O^{14} O^{24} O^{34} O^{15} O^{25} O^{35} O^{45}, \quad (2)$$

where ( $O^{45}$ , giving sterile-sterile neutrino mixing, is not shown)

$$\begin{aligned} O^{23} &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & c_{23} & s_{23} & 0 & 0 \\ 0 & -s_{23} & c_{23} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} & O^{13} &= \begin{pmatrix} c_{13} & 0 & s_{13} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -s_{13} & 0 & c_{13} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \\ O^{12} &= \begin{pmatrix} c_{12} & s_{12} & 0 & 0 & 0 \\ -s_{12} & c_{12} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} & O^{14} &= \begin{pmatrix} c_\alpha & 0 & 0 & s_\alpha & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -s_\alpha & 0 & 0 & c_\alpha & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \\ O^{24} &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & c_\alpha & 0 & s_\alpha & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -s_\alpha & 0 & c_\alpha & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} & O^{34} &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & c_\alpha & s_\alpha & 0 \\ 0 & 0 & -s_\alpha & c_\alpha & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \\ O^{15} &= \begin{pmatrix} c_\beta & 0 & 0 & 0 & s_\beta \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -s_\beta & 0 & 0 & 0 & c_\beta \end{pmatrix} & O^{25} &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & c_\beta & 0 & 0 & s_\beta \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & -s_\beta & 0 & 0 & c_\beta \end{pmatrix} \\ O^{35} &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & c_\beta & 0 & s_\beta \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -s_\beta & 0 & c_\beta \end{pmatrix} \end{aligned}$$

$\mathcal{P}(\nu_\mu \rightarrow \nu_e)$  is obtained from the 5x5 U matrix and the neutrino mass differences  $\delta m_{ij}^2 = m_i^2 - m_j^2$  for a neutrino beam with energy  $E$  and baseline  $L$  by

$$\mathcal{P}(\nu_\mu \rightarrow \nu_e) = \text{Re} \left[ \sum_{i=1}^5 \sum_{j=1}^5 U_{1i} U_{1j}^* U_{2i}^* U_{2j} e^{-i(\delta m_{ij}^2/E)L} \right], \quad (3)$$

an extension of the 4x4[7, 8] theory with one sterile neutrino, which used the 3x3 formalism of Ref[5], to a 5x5 matrix formalism. From Eq(2), multiplying the nine 5x5  $O$  matrices, we obtain the matrix U. With  $\delta_{CP}=0$ ,  $U_{ij}^* = U_{ij}$ , so we only need  $U_{1j}, U_{2j}$ . The active neutrino mixing parameters[6] are  $c23 = s23 = .7071, c13 = .989, s13 = .15, c12 = .83, s12 = .56$ .

$$\begin{aligned} U_{11} &= .821ca \ cb \\ U_{12} &= (.554ca - .821sa^2)cb - .821ca \ sb^2 \\ U_{13} &= (.15ca - .554sa^2 - .821ca \ sa^2)cb - (.554ca - .821sa^2)sb^2 \\ &\quad + .821ca \ cb \ sb^2 \\ U_{14} &= cb(.15sa + .554ca \ sa + .821ca^2 \ sa) - .821ca \ cb^2 \ sb^2 \\ &\quad - (.554ca - .821sa^2)cb \ sb^2 - (.15ca - .554sa^2 - .821casa^2)sb^2 \\ U_{15} &= .821ca \ sb \ cb^3 + (.15sa + .554ca \ sa + .821ca^2 \ sa)sb \\ &\quad + (.554ca - .821sa^2)cb^2 \ sb + (.15ca - .554sa^2 - .821ca \ sa^2)cb \ sb \\ U_{21} &= -.484ca \ cb \\ U_{22} &= (.527ca + .484sa^2)cb + .484ca \ sb^2 \\ U_{23} &= (.699ca - .527sa^2 + .484ca \ sa^2)cb - (.527ca + .484sa^2)sb^2 + .484ca \ cb \ sb^2 \\ U_{24} &= cb(.699sa + .527ca \ sa - .484ca^2 \ sa) + .484ca \ cb^2 \ sb^2 \\ &\quad - (.527ca + .484sa^2)cb \ sb^2 - (.699ca - .527sa^2 + .484ca \ sa^2)sb^2 \\ U_{25} &= -.484ca \ sb \ cb^3 + (.699sa + .527ca \ sa - .484ca^2 \ sa)sb \\ &\quad + (.527ca + .484sa^2)cb^2 \ sb + (.699ca - .527sa^2 + .484ca \ sa^2)cb \ sb \end{aligned} \quad (4)$$

The active neutrino mass differences are  $\delta m_{12}^2 = m_2^2 - m_1^2 = 7.6 \times 10^{-5}(eV)^2$ ,  $\delta m_{13}^2 = m_3^2 - m_1^2 \simeq \delta m_{23}^2 = 2.4 \times 10^{-3}(eV)^2$ . From Ref[4] the first sterile-active mass difference  $= \delta m_{4i}^2 = m_4^2 - m_i^2 \simeq 1.75 (eV)^2$ , with  $i=1,2,3$  for active neutrinos; and  $s_\alpha^2 \simeq 2.6 \times 10^{-2}$ , or  $\alpha \simeq 9.2^\circ$ . Because of the difficulty in the analysis we assume that  $\delta m_{4i}^2 = \delta m_{5i}^2$ ,  $\delta m_{54}^2 = 0$  and  $\alpha = \beta$ . Note that the sterile-active mixing angle used in Refs[7, 8] was also  $9.2^\circ$ .

### 3 $\mathcal{P}(\nu_\mu \rightarrow \nu_e)$ For Two Sterile Neutrinos

With the mass differences  $\delta m_{12}^2$ ,  $\delta m_{13}^2$ ,  $\delta m_{23}^2$ ,  $\delta m_{4i}^2$ ,  $\delta m_{5i}^2$ ,  $\delta m_{54}^2$  given above, we define  $\delta = \delta m_{12}^2/2E$ ,  $\Delta = \delta m_{13}^2/2E$ ,  $\gamma = \delta m_{4i}^2/2E$ ,  $\lambda = \delta m_{5i}^2/2E$ ,  $\kappa = \delta m_{54}^2/2E$ .

$$\begin{aligned} \mathcal{P}(\nu_\mu \rightarrow \nu_e) = & \text{Re}[U_{11}U_{21}(U_{11}U_{21} + U_{12}U_{22}e^{-i\delta L} + U_{13}U_{23}e^{-i\Delta L} + \\ & U_{14}U_{24}e^{-i\gamma L} + U_{15}U_{25}e^{-i\lambda L}) + \\ & U_{12}U_{22}(U_{11}U_{21}e^{-i\delta L} + U_{12}U_{22} + U_{13}U_{23}e^{-i\Delta L} + \\ & U_{14}U_{24}e^{-i\gamma L} + U_{15}U_{25}e^{-i\lambda L}) + U_{13}U_{23}(U_{11}U_{21}e^{-i\Delta L} + U_{12}U_{22}e^{-i\Delta L} \\ & + U_{13}U_{23} + U_{14}U_{24}e^{-i\gamma L} + U_{15}U_{25}e^{-i\lambda L}) + U_{14}U_{24}((U_{11}U_{21} + U_{12}U_{22} \\ & + U_{13}U_{23})e^{-i\gamma L} + U_{14}U_{24} + U_{15}U_{25}e^{-i\kappa L}) \\ & + U_{15}U_{25}((U_{11}U_{21} + U_{12}U_{22} + U_{13}U_{23})e^{-i\lambda L} + U_{14}U_{24}e^{-i\kappa L} + U_{15}U_{25})] \end{aligned} \quad (5)$$

From Eq(5)

$$\begin{aligned} \mathcal{P}(\nu_\mu \rightarrow \nu_e) = & U_{11}^2 U_{21}^2 + U_{12}^2 U_{22}^2 + U_{13}^2 U_{23}^2 + U_{14}^2 U_{24}^2 + U_{15}^2 U_{25}^2 + \\ & 2U_{11}U_{21}U_{12}U_{22}\cos\delta L + \\ & 2(U_{11}U_{21}U_{13}U_{23} + U_{12}U_{22}U_{13}U_{23})\cos\Delta L + \\ & 2U_{14}U_{24}(U_{11}U_{21} + U_{12}U_{22} + U_{13}U_{23})\cos\gamma L + \\ & 2U_{15}U_{25}(U_{11}U_{21} + U_{12}U_{22} + U_{13}U_{23})\cos\lambda L + \\ & 2U_{14}U_{24}U_{15}U_{25}\cos\kappa L. \end{aligned} \quad (6)$$

From the discussion below Eq(4),  $\alpha \simeq \beta \simeq 9.2^\circ$ , with  $sa = sb \simeq 0.16$  and  $ca = cb \simeq 0.9871$ , which are used to determine  $U_{1j}, U_{2j}$  in Eq(4).

In the figure below, the results of the two sterile neutrinos on  $\mathcal{P}(\nu_\mu \rightarrow \nu_e)$  using Eq(6) and the parameters obtained from Ref[4] are shown for four experimental neutrino oscillation experiments.

The figure also shows  $\mathcal{P}(\nu_\mu \rightarrow \nu_e)$  with  $\alpha = \beta = 0^\circ$ , giving the results of a recent 3x3 S-matrix calculation[9] to compare to the results with two sterile neutrinos.

Using Eq(6), one finds  $\mathcal{P}(\nu_\mu \rightarrow \nu_e)$

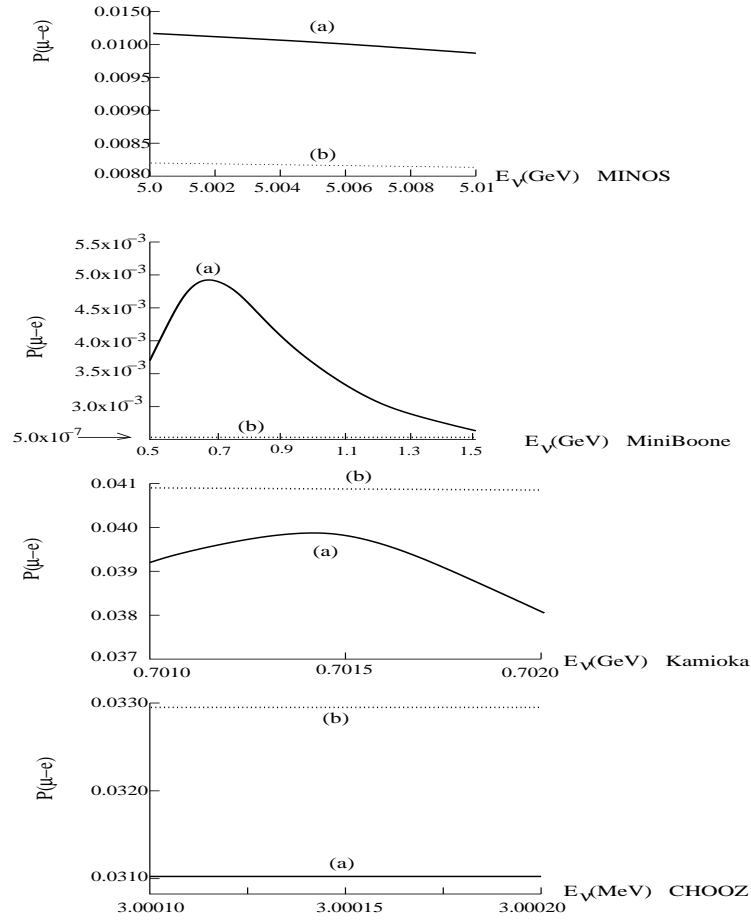


Figure 1:  $\mathcal{P}(\nu_\mu \rightarrow \nu_e)$  for MINOS(L=735 km), MiniBooNE(L=500m), JHF-Kamioka(L=295 km), and CHOOZ(L=1.03 km). (a) solid for  $\alpha = \beta = 9.2^\circ$ ; (b) dashed curve for  $\alpha = \beta = \gamma = 0^\circ$  giving the 3x3 result.

## 4 $\mathcal{P}(\nu_\mu \rightarrow \nu_e)$ For One Sterile Neutrino

As mentioned above, in our previous articles on  $\mathcal{P}(\nu_\mu \rightarrow \nu_e)$  with one sterile neutrino[7, 8] we used the sterile neutrino-active neutrino mixing angle =  $9.2^\circ$  from Refs[1, 2, 3], while from the recent Ref[4] the sterile neutrino-active neutrino mixing angle  $\alpha = \theta_{i4} \simeq 9.2^\circ$ , with  $sa \simeq 0.16$  and  $ca \simeq 0.9871$  for the first sterile neutrino. Also, in Refs[7, 8] we used  $\delta m_{4i}^2 = m_4^2 - m_i^2 \simeq 0.9 \text{ (eV)}^2$ , while from Ref[4]  $\delta m_{4i}^2 \simeq 1.75 \text{ (eV)}^2$ , which also changes the estimate of  $\mathcal{P}(\nu_\mu \rightarrow \nu_e)$  with one sterile neutrino.

As discussed in Ref[7] the transition probability  $\mathcal{P}(\nu_\mu \rightarrow \nu_e)$ , assuming  $\delta_{CP} = 0$  giving  $U_{ij}^* = U_{ij}$ , is

$$\begin{aligned} \mathcal{P}(\nu_\mu \rightarrow \nu_e) = & U_{11}^2 U_{21}^2 + U_{12}^2 U_{22}^2 + U_{13}^2 U_{23}^2 + \\ & U_{14}^2 U_{24}^2 + 2U_{11}U_{21}U_{12}U_{22}\cos\delta L + \\ & 2(U_{11}U_{21}U_{13}U_{23} + U_{12}U_{22}U_{13}U_{23})\cos\Delta L + \\ & 2U_{14}U_{24}(U_{11}U_{21} + U_{12}U_{22} + U_{13}U_{23})\cos\gamma L, \end{aligned} \quad (7)$$

with the parameters defined above.

Using  $c_{12} = .83$ ,  $s_{12} = .56$ ,  $s_{23} = c_{23} = .7071$ , and  $s_{13} = .15$ , (with  $s_{ij}, c_{ij} = \sin\theta_{ij}, \cos\theta_{ij}$ ),

$$\begin{aligned} U_{11} &= .822c_\alpha \\ U_{12} &= .554c_\alpha - .821s_\alpha^2 \\ U_{13} &= -.821s_\alpha^2 c_\alpha - .554s_\alpha^2 + .15c_\alpha \\ U_{14} &= .821s_\alpha c_\alpha^2 + .554s_\alpha c_\alpha + .15s_\alpha \\ U_{21} &= -.484c_\alpha \\ U_{22} &= .484s_\alpha^2 + .527c_\alpha \\ U_{23} &= .699c_\alpha - (-.484s_\alpha c_\alpha + .527s_\alpha)s_\alpha \\ U_{24} &= -.484s_\alpha c_\alpha^2 + .527s_\alpha c_\alpha + .699s_\alpha, \end{aligned} \quad (8)$$

with  $\alpha$  the sterile-active neutrino mixing angle,  $s_\alpha, c_\alpha = \sin(\alpha), \cos(\alpha)$

In Figure 2 we compare  $\mathcal{P}(\nu_\mu \rightarrow \nu_e)$  with one sterile neutrino using the sterile-active neutrino mixing angle of  $9.2^\circ$ [4] ( $\sin(\alpha) \simeq 0.16$ ), and  $\delta m_{4i}^2 \simeq 1.75 \text{ (eV)}^2$  vs  $\delta m_{4i}^2 \simeq 0.9 \text{ (eV)}^2$  in Ref[8]; and  $\mathcal{P}(\nu_\mu \rightarrow \nu_e)$  with no sterile neutrino. Note the results are different from those in Ref[8] because of the mass differences.

In Figure 2 the solid curves are estimates of  $\mathcal{P}(\nu_\mu \rightarrow \nu_e)$  using the parameters from Ref[4] for one sterile neutrino, while the dashed curves are  $\mathcal{P}(\nu_\mu \rightarrow \nu_e)$  with only active neutrinos.

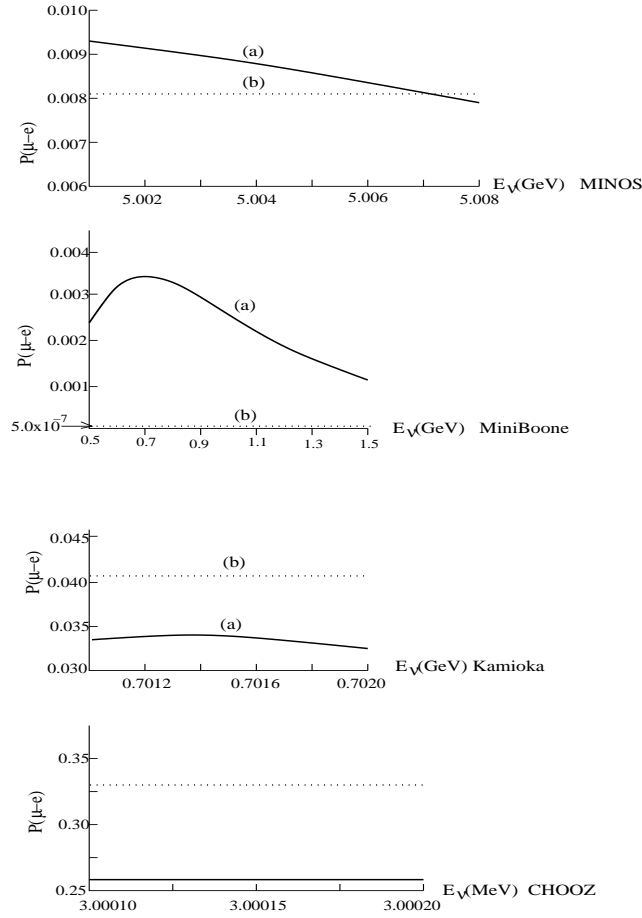


Figure 2:  $\mathcal{P}(\nu_\mu \rightarrow \nu_e)$  for MINOS(L=735 km), MiniBooNE(L=500m), JHF-Kamioka(L=295 km), and CHOOZ(L=1.03 km). (a) solid for  $\alpha = 9.2^\circ$ ; (b) dashed curve for  $\alpha = 0^\circ$ , giving the 3x3 result.

## 5 Conclusions

From Figure 1 we note that even with the small mixing angles,  $\alpha = \beta = 9.2^\circ$ , obtained from the analyses given in Ref[4] there is significant difference between our 5x5 and the earlier 3x3 prediction for  $\mathcal{P}(\nu_\mu \rightarrow \nu_e)$ , given by  $\alpha = \beta = 0^\circ$ . Also, from Figure 2, for one sterile neutrino  $\mathcal{P}(\nu_\mu \rightarrow \nu_e)$  differs significantly for  $\alpha = 9.2^\circ$ ,  $\delta m_{4i}^2 \simeq 1.75(eV)^2$  given in Ref[4] compared to  $\alpha = 9.2^\circ$ ,  $\delta m_{4i}^2 \simeq 0.9(eV)^2$  used in Ref[8].

Therefore in future neutrino oscillation experiments the effect of two sterile neutrinos should be measured. Also the value of the sterile-active neutrino mixing angle might be more accurately determined in the near future.

## Acknowledgements

This work was carried out in part while LSK was a visitor at Los Alamos National Laboratory, Group P25. The author thanks Dr. William Louis for several discussions and information concerning neutrino oscillation experiments and the analysis to determine the mixing angles.

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